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The statistics of electron counting

I M Every

Department of Physics, Monash University, Clayton, Victoria 3168, Australia

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Abstract. The probability $P(n, t_1, t_2)$ of counting n electrons in the time interval from t_1 to t_2 is derived from a quantum-electrodynamic treatment of an electron multiplier. Although the expressions obtained are similar to those obtained by Glauber for photons there remain fundamental differences. The interest of these results for existing and proposed experiments on electron beams is outlined.

1. Introduction

The concepts of photon counting statistics and photon coherence have aroused much theoretical and experimental interest in recent years, with the demonstration of the Hanbury Brown–Twiss effect (Hanbury Brown and Twiss 1957a,b), the advent of sophisticated photon counting experiments (Troup and Lyons 1971, Arecchi *et al* 1966, Kelly and Blake 1971, 1972, Jakeman *et al* 1971), and particularly with the tremendous clarification and increased understanding achieved by a quantum-mechanical treatment of optical coherence (Glauber 1964, Kelly and Kleiner 1964). For reviews of these developments see Mandel and Wolf (1965), Klauder and Sudarshan (1968), and Troup (1972). Unfortunately these concepts have not been applied to electron beams except in a loose way based on rough analogies with classical optics (Hibi and Takahashi 1969, Klemperer 1972, Heidenreich 1964) and the aim of this paper is to treat electron counting statistics and coherence from the viewpoint of quantum electrodynamics, this being all the more necessary because of basic differences between the mechanisms of photon and electron detection. In spite of these differences however, it will be shown that the resulting expressions for the statistics of electron counting are very similar to those for the photon case, but with several important restrictions. The fact that electron field operators obey anticommutation rules plays an important role, forbidding the existence of non-trivial, fully coherent states (Bowring *et al* 1971), and preventing the use of a theorem which is very powerful in quantum optics: the optical equivalence theorem.

2. Calculation of the detection probability

2.1. The detection process

The choice of a detection process for electrons is (as in the photon case) not unique, but perhaps the most common detector which is capable of resolving individual electrons of low–medium energy is the electron multiplier. Unlike photon detection however, it is not evident that electron detection in the electron multiplier can be treated rigorously

using a dipole approximation, and moreover the detection system involves the same field as the detected beam. The detection process is not the simple absorption diagram of the photoelectric effect but is probably to lowest order the second-order diagram of bound Møller scattering between electrons in the beam and electrons in the detector. These are in general in the external field of the nuclei or perhaps as a further approximation in their own self-consistent field. The term in the scattering operator for this diagram is

$$S_{\mathrm{BMS}} = \int \mathrm{d}^3\boldsymbol{r}_1 \int \mathrm{d}^3\boldsymbol{r}_2 \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \sum_{\substack{i,j,k,l \\ \mu,\nu}} \bar{\psi}_i^-(x_1)\bar{\psi}_j^-(x_2)\psi_k^+(x_1)\psi_l^+(x_2)$$
$$\times \gamma_{ik}^{\mu}\gamma_{jl}^{\nu}\langle 0|\{A_{\mu}^-(x_1)A_{\nu}^+(x_2)\}_+|0\rangle \tag{1}$$

where x_i stands for the four-vector $(\boldsymbol{r}_i, t_i)$.

In order to simplify the calculations it is necessary to assume the distinguishability or separateness of (i) the incoming beam (B), (ii) the electrons in the detector (P), (iii) the scattered electrons (S) and (iv) the excited electrons (emitted from the cathode) (E), that is, it is necessary to assume that the one-particle eigenstates of the external field plus electron field hamiltonian (solutions of the Dirac equation) can be divided into these four types. Expanding the electron field operator in terms of these one-particle functions $\Phi_j^{\alpha}(\boldsymbol{r})$ with energy eigenvalues $\hbar\omega(\alpha)$ gives:

$$\psi_j^+(\boldsymbol{r}, t) = \sum_{\alpha} c_j^{\alpha}\Phi_j^{\alpha}(\boldsymbol{r})\exp(\mathrm{i}\omega(\alpha)t)$$
$$= \psi_j^{+(\mathrm{B})} + \psi_j^{+(\mathrm{D})} + \psi_j^{+(\mathrm{S})} + \psi_j^{+(\mathrm{E})} \tag{2}$$

where j is the spinor index and c_j^{α} is the annihilation operator of the jth spinor component of an electron in the state corresponding to $\Phi_j^{\alpha}(\boldsymbol{r})$ and

$$\psi_j^{+(\mathrm{B,D,S,E})} \equiv \sum_{\alpha\in\mathrm{B,D,S,E}} c_j^{\alpha}\Phi_j^{\alpha}(\boldsymbol{r})\exp(\mathrm{i}\omega(\alpha)t).$$

Substituting (2) into (1) yields 256 terms in the expansion for S_{BMS}. However, by the choice of the regions in state space (B, D, S and E) and the assumption that incident, detector, scattered and excited electron states lie predominantly in their assigned regions, this number reduces to two. If the state space cannot be divided into such regions, then the calculation will be enormously complicated by interference effects, between beam and scattered electrons for example.

2.2. The first-order detection probability

Let $\{|\epsilon\rangle\}$ be the set of states of the electron field (in the region E) which are considered as having been detected or excited in some sense. These will be the states in which an electron is travelling from the cathode to the second electrode in the cascade. Treating only this first step in the cascade quantum mechanically, the probability of detecting an electron in a small time interval, Δt, is proportional to, to first order,

$$P^{(1)}(\Delta t) = \sum_{\epsilon} \langle \mathrm{i}|S_{\mathrm{BMS}}^{\dagger}|\epsilon\rangle\langle\epsilon|S_{\mathrm{BMS}}|\mathrm{i}\rangle. \tag{3}$$

The initial state $|\mathrm{i}\rangle$ is taken to be a product of detector and beam states (appropriately antisymmetrized), that is, they are assumed to be independent or uncorrelated at $t = 0$. This will be realized if the two have not interacted at this time or if there is some interaction within the detector which causes any correlation which might have arisen to be

destroyed in a relaxation process. Let the initial state of the beam be denoted $|\Psi\rangle$ and that of the detector $|\phi_0\rangle$, with the scattered and excited regions in their ground states $|0_S\rangle, |0_E\rangle$.

Substituting the appropriate terms of (1) into (3), rearranging the variables of integration and factorizing into expectations in the four regions B, D, S, E, yields the result below. (Antisymmetrization of the initial state may be ignored, as rearrangement of the order of the states necessitates the same rearrangement of operators and thus an odd permutation leads to no resultant change in sign.)

$$P^{(1)}(\Delta t) = \sum_{\substack{jj'\\ii'}} \int\int\int\int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_3\,\mathrm{d}^4x_4 \langle\Psi|\psi_j^{-(B)}(x_2)\psi_{j'}^{+(B)}(x_4)|\Psi\rangle$$
$$\times \langle\phi_0|\psi_i^{-(D)}(x_3)\psi_{i'}^{+(D)}(x_1)|\phi_0\rangle \{{}^{ii'}W_{jj'}^{(1)}(t,\boldsymbol{r}) + {}^{ii'}W_{jj'}^{(2)}(t,\boldsymbol{r}) - {}^{ii'}W_{jj'}^{(3)}(t,\boldsymbol{r})$$
$$- {}^{ii'}W_{jj'}^{(4)}(t,\boldsymbol{r})\} D(x_1-x_2)D(x_3-x_4) \tag{4}$$

where

$${}^{ii'}W_{jj'}^{(1)}(\boldsymbol{r},t) = \sum_{\substack{\mu\nu kl\epsilon\\ \mu'\nu'k'l'}} (\bar\gamma^\mu\gamma^4)_{li}(\bar\gamma^\nu\gamma^4)_{kj}(\gamma^4\gamma^{\mu'})_{l'i'}(\gamma^4\gamma^{\nu'})_{k'j'}\langle 0_S|\psi_l^{+(S)}(x_1)\psi_{l'}^{-(S)}(x_3)|0_S\rangle$$
$$\times \langle 0_E|\psi_k^{+(E)}(x_2)|\epsilon\rangle\langle\epsilon|\psi_{k'}^{-(E)}(x_4)|0_E\rangle \tfrac{1}{4}\delta_{\mu\nu}\delta_{\mu'\nu'}$$
$${}^{ii'}W_{jj'}^{(2)}(\boldsymbol{r},t) = \sum_{\substack{\mu\nu kl\epsilon\\ \mu'\nu'k'l'}} (\bar\gamma^\mu\gamma^4)_{ki}(\bar\gamma^\nu\gamma^4)_{lj}(\gamma^4\gamma^{\mu'})_{k'i'}(\gamma^4\gamma^{\nu'})_{l'j'}\langle 0_S|\psi_l^{+(S)}(x_2)\psi_{l'}^{-(S)}(x_4)|0_S\rangle$$
$$\times \langle 0_E|\psi_k^{+(E)}(x_1)|\epsilon\rangle\langle\epsilon|\psi_{k'}^{-(E)}(x_3)|0_E\rangle \tfrac{1}{4}\delta_{\mu\nu}\delta_{\mu'\nu'}$$
$${}^{ii'}W_{jj'}^{(3)}(\boldsymbol{r},t) = \sum_{\substack{\mu\nu kl\epsilon\\ \mu'\nu'k'l'}} (\bar\gamma^\mu\gamma^4)_{li}(\bar\gamma^\nu\gamma^4)_{kj}(\gamma^4\gamma^{\mu'})_{k'i'}(\gamma^4\gamma^{\nu'})_{l'j'}\langle 0_S|\psi_l^{+(S)}(x_1)\psi_{l'}^{-(S)}(x_4)|0_S\rangle$$
$$\times \langle 0_E|\psi_k^{+(E)}(x_2)|\epsilon\rangle\langle\epsilon|\psi_{k'}^{-(E)}(x_3)|0_E\rangle \tfrac{1}{4}\delta_{\mu\nu}\delta_{\mu'\nu'}$$
$${}^{ii'}W_{jj'}^{(4)}(\boldsymbol{r},t) = \sum_{\substack{\mu\nu kl\epsilon\\ \mu'\nu'k'l'}} (\bar\gamma^\mu\gamma^4)_{ki}(\bar\gamma^\nu\gamma^4)_{lj}(\gamma^4\gamma^{\mu'})_{l'i'}(\gamma^4\gamma^{\nu'})_{k'j'}\langle 0_S|\psi_l^{+(S)}(x_2)\psi_{l'}^{-(S)}(x_3)|0_S\rangle$$
$$\times \langle 0_E|\psi_k^{+(E)}(x_1)|\epsilon\rangle\langle\epsilon|\psi_{k'}^{-(E)}(x_4)|0_E\rangle \tfrac{1}{4}\delta_{\mu\nu}\delta_{\mu'\nu'}$$

also denote

$${}^{ii'}W_{jj'}^{(a)} \equiv {}^{ii'}W_{jj'}^{(1)} + {}^{ii'}W_{jj'}^{(2)}$$
$${}^{ii'}W_{jj'}^{(b)} \equiv {}^{ii'}W_{jj'}^{(3)} + {}^{ii'}W_{jj'}^{(4)}$$
$$S_{jj'}^{(F)}(x_2,x_4) \equiv \sum_{ii'} \int_0^{\Delta t}\int_0^{\Delta t} \mathrm{d}^4x_1\,\mathrm{d}^4x_3 \langle\phi_0|\psi_i^{-(D)}(x_1)\psi_{i'}^{+(D)}(x_3)|\phi_0\rangle$$
$$\times \{{}^{ii'}W_{jj'}^{(a)}(r_s,t_s) + {}^{ii'}W_{jj'}^{(b)}(r_s,t_s)\} D(x_1-x_2)D(x_3-x_4). \tag{5}$$

Using these notations

$$P^{(1)}(\Delta t) = \sum_{jj'} \int_0^{\Delta t}\int_0^{\Delta t} S_{jj'}^{(F)}(x_2,x_4,\Delta t)\langle\psi_j^{-(B)}(x_2)\psi_{j'}^{+(B)}(x_4)\rangle\,\mathrm{d}^4x_2\,\mathrm{d}^4x_4. \tag{6}$$

The effects of two classes of interactions (which have been neglected in the treatment above) must be considered. Namely (i) interactions between electrons in the detector and (ii) interactions between electrons in the beam. Since the states of the detector and

possibly even of the beam are taken to be stationary a large part of (i), and to a lesser extent (ii), can be treated by considering an external self-consistent field produced by the electrons, which just adds to the external field of the nuclei and can be treated in the same way. The effects of the interactions (ii) will be minimal if the beam is dilute everywhere and not of low energy (ie >100 eV).

The expression (4) for $P^{(1)}(\Delta t)$ depends implicitly on the functions $\Phi_j^z(\boldsymbol{r})$ which in general are not plane waves. Because it is convenient to deal with plane waves outside the detector, it is necessary to find solutions in the detector which match up with the plane waves at the boundary. If the detector can be considered as a potential well for this purpose, it is simply a matter of matching plane waves inside the detector with plane waves of different momentum outside, but the general problem is more difficult. This procedure is unnecessary for photon detectors since, to an excellent approximation, the photon does not interact with the electromagnetic field in the detector.

2.3. Higher-order counting rates and statistics

In order to get a fully normalized probability distribution for the probability $P(n, t_1, t_2)$ of counting n counts in a given time interval, $[t_1, t_2]$, it is necessary to take the S matrix expansion as a non-truncated infinite series. As this is on the whole impractical, other approaches or simplifying assumptions have to be made, such as those made by Glauber (1964), Kelly and Kleiner (1964), and Rocca (1972) for the photon detection case. Applying such assumptions to electron detection gives

$$P(n, t_1, t_2) = \sum_{m=0}^{\infty} \int_{t_1}^{t_2} \mathrm{d}^4 x_1 \ldots \int_{t_1}^{t_2} \mathrm{d}^4 x_{m+n} \int_{t_1}^{t_2} \mathrm{d}^4 x_1' \ldots \int_{t_1}^{t_2} \mathrm{d}^4 x_{m+n}' \frac{1}{m!n!}$$
$$\times \sum_{j(1),j(2)\ldots} \sum_{j'(1),j'(2)\ldots} \prod_{i=1}^{m+n} S^{(\mathrm{F})}_{j(i)j'(i)}(x_i, x_i', t_1, t_2) \left\langle \Psi \middle| \prod_{i=1}^{m+n} \psi^{-(\mathrm{B})}_{j(i)}(x_i) \prod_{i=m+n}^{1} \psi^{+(\mathrm{B})}_{j'(i)}(x_i') \middle| \Psi \right\rangle \qquad (7)$$

where the spinor indices j, j' are dependent on i in each of the three groups of products.

All of these assumptions involve considering only products of the lowest-order diagram, in this case the bound Møller scattering diagram, and hence ignoring any processes which occur only at higher order. These processes are possibly more important for electron detection than photon detection. In both these cases such processes are vital for amplifying the effect of the incoming particle, but it is probably a good approximation to treat this separately, in the same way that such macroscopic effects as dead time are treated (Mehta 1970, Goldanskii *et al* 1962, Bedard 1967). If any of these processes, however, has a time spread which is greater than the period of the beats which occur, then these, and hence any interesting statistical properties of the beam, will not be revealed.

3. The sensitivity function

The expressions (7) for $P(n, t_1, t_2)$ and (6) for $P^{(1)}(\Delta t)$ are at least superficially similar to the corresponding formulae for photon counting (Kelly and Kleiner 1964), where $S^{(\mathrm{P})}_{\mu\nu}(\boldsymbol{r}_2, \boldsymbol{r}_4, t_2 - t_4)$ can be interpreted as a sensitivity function. If $S^{(\mathrm{F})}_{jj'}(x_2, x_4, \Delta t)$ is to be interpreted as such a sensitivity function it must possess at least two properties of $S^{(\mathrm{P})}_{\mu\nu}$: (i) it must be a function of $\boldsymbol{r}_2$, $\boldsymbol{r}_4$ and $t_2 - t_4$ only; and (ii) it must be independent of the time of observation Δt, to first order. By studying the invariance properties of $S^{(\mathrm{F})}_{jj'}$

under translations in time and using an expansion for the vacuum contraction $D(x_1, x_2)$ (Akhieser and Berestetskii 1965, p 515) the two properties can be shown to hold if

$$\Delta t \gg R/C \tag{8}$$

where C is the speed of light and R is the 'range of interaction' where $q_e/(4\pi\epsilon_0 R)$ is the lowest excitation energy. This restriction does not occur for photons because in the photoelectric effect the excited electron is always created at the same time and place as the photon is destroyed.

Now consider the dependence of $S_{jj'}^{(F)}$ on the spatial coordinates $\boldsymbol{r}_2$ and $\boldsymbol{r}_4$, and the effect of this on the detection of beats in the case where the one-particle solutions $\Phi_j^{\alpha}(\boldsymbol{r})$ in the region D are localized, that is, they can be chosen so that each one takes on a non-negligible value only inside a finite volume (with a maximum dimension, d, say). This would be true for instance if the detection system consisted of an array of non-interacting 'atoms' (ie atoms, molecules or ions), where $\Phi_j^{\alpha}(\boldsymbol{r})$ would then represent the wavefunctions for the orbitals of the 'atoms'. From this we have

$$\langle \phi_0 | \psi_i^{-(D)}(x_1) \psi_{i'}^{+(D)}(x_3) | \phi_0 \rangle \simeq 0 \qquad \text{unless } |\boldsymbol{r}_1 - \boldsymbol{r}_3| < d.$$

Combining this with (10) gives

$$J \simeq 0 \qquad \text{unless } |\boldsymbol{r}_2 - \boldsymbol{r}_4| < R + d$$

where J is the integrand of (5). If these localized volumes are separated by a distance greater than $R+d$, then this corresponds to the case where the detector can be divided up into separate 'atoms', which are non-interacting and with which an electron can interact only one at a time. This is probably only a fair approximation for solid state detectors, but it would appear to be a very good approximation for an ionization type detector consisting of an atomic or molecular gas.

If this dimension $R+d$ is small compared with the electron wavelength, λ_e, (there may be physical difficulties in this) then $S_{jj'}^{(F)}(t_2 - t_4, \boldsymbol{r}_2, \boldsymbol{r}_4)$ may be approximated by $S_{jj'}^{(F)}(t_2 - t_4, \boldsymbol{r}_2)\delta(\boldsymbol{r}_2 - \boldsymbol{r}_4)$ and the first-order detection probability may be written in the form

$$P^{(1)}(\Delta t) = \sum_{jj'} \int \mathrm{d}^3\boldsymbol{r} \int\int_0^{\Delta t} \mathrm{d}t_2 \, \mathrm{d}t_4 S_{jj'}^{(F)}(t_2 - t_4, \boldsymbol{r}) \langle \psi_j^{-(B)}(\boldsymbol{r}, t_2) \psi_{j'}^{+(B)}(\boldsymbol{r}, t_4) \rangle. \tag{9}$$

This is basically similar to the formula for photon detection derived by such authors as Glauber (1964) who implicitly rely on the above considerations. The simplicity of the expression (9) for $P^{(1)}(\Delta t)$ would make it preferable to study this case experimentally, but this imposes restrictions on the energy (E eV) of the electrons and on the effective size of the 'atoms' ($R+d$), which may be difficult to satisfy. Explicitly (for non-relativistic electrons),

$$R + d < \lambda_e \simeq 10^{-9} E^{-1/2}.$$

Thus for an atom of effective diameter 10^{-8} m the electron energy must be below 0·01 eV.

Alternatively consider $\lambda_e \leqslant R+d$. Dividing $P^{(1)}(\Delta t)$ into a sum over atoms and expanding the operators $\psi_j^{\pm(B)}(x)$ in terms of the $\Phi_j^{\alpha}(\boldsymbol{r})$ and the time dependent creation and annihilation operators

$${}^{j}a_{\alpha} \equiv {}^{j}c_{\alpha} \exp(-\mathrm{i}\omega(\alpha)t)$$

yields the following result:

$$P^{(1)}(\Delta t) = \sum_{jj'} \sum_{m} \sum_{k_1 k_2} \int\int_0^{\Delta t} s_{jj'}^{(m)}(t_2 - t_4, k_1, k_2) \langle {}^{j}a_{k_1}^{\dagger}(t_2)\, {}^{j'}a_{k_2}(t_4) \rangle \tag{10}$$

where

$$s_{jj'}^{(m)}(t_2 - t_4, k_1, k_2) \equiv \int\int_{\text{volume '}m\text{'}} \mathrm{d}^3 \boldsymbol{r}_2\, \mathrm{d}^3 \boldsymbol{r}_4 S_{jj'}^{(\mathrm{F})}(t_2 - t_4, \boldsymbol{r}_2, \boldsymbol{r}_4) \bar{\phi}_{j'}^{k_1}(\boldsymbol{r}_2) \Phi_{j}^{k_2}(\boldsymbol{r}_4). \tag{11}$$

If the 'atoms' are identical in type and in their environment then $s_{jj'}^{(m)}$ will not depend on m. The result (10) differs from the corresponding expression derived from (9) by a dependence on k_1 and k_2 together, quite apart from the general frequency response of the detector. This, in general, can be a very strong dependence which will alter the relationships between the counting statistics and the line shape, making them much more complicated than for the usual photon case.

In either case, $\lambda_e > R + d$ or $\lambda_e \leqslant R + d$, as the total effective dimension l of the detector becomes comparable with, or larger than, the wavelength of the beats, then these will become averaged out to an extent which depends critically on the appropriate dimension of the detector. If any interesting statistical effects are to be observed, then this effective dimension will have to be constant and uniform to within the beat wavelength. For a nearly parallel beam with mean energy E eV and a small energy spread ΔE eV this means

$$\Delta l \Delta E < 2 \times 10^{-9} E^{1/2}. \tag{12}$$

For example with a 10 keV beam and a detector with a fluctuation in dimension of 0·02 mm, the energy spread must be less than 0·01 eV.

Finally, consider the dependence of the sensitivity function on the spin states of both the detector and the incident beam. If we assume that there is no discrimination between the spins of the excited (secondary) electrons then it can be shown that ${}^{ii'}W_{jj'}^{(\mathrm{a})}$ is diagonal in both ii' and jj' and hence that the corresponding term in the first-order detection probability is completely insensitive to the spin of both beam and detector electrons. On the other hand ${}^{ii'}W_{jj'}^{(\mathrm{b})}$ is diagonal in ij' and $i'j$ and hence describes the spin-dependent sensitivity of the detector. If electrons in the detector have a preferred spin then the beam electrons of the same type of spin are detected with reduced sensitivity. If the detector is completely unpolarized then the detector is insensitive to spin.

4. Conclusions

The formulae for $P(n, t_1, t_2)$ derived in the previous sections provide a general method of calculating from a given quantum state the results of experiments designed to exhibit the statistical properties of electron beams. Although the formulae appear similar to those derived for photon beams there are a number of important differences which will be of interest for future experiments with electron beams. The relation $R + d < \lambda_e$ appears to be extremely restrictive because of the shortness of electron wavelengths and the range of the Coulomb interaction. It is thus unlikely that the idealization of a point detector (which is applicable in photon optics) will be realizable and it would be necessary to use the more general expression (equation (7)). The expressions for $P(n_1, t_1, t_2)$ also differ markedly from those for the photon case in that the operators involved (ψ^- and ψ^+)

obey anticommutation rather than commutation relations and satisfy the Dirac rather than the Klein–Gordan equation. First-order coherence may be defined, following the definition for photon coherence,

$$g_{ij}(x_1, x_2) = \frac{\langle \psi_i^-(x_1)\psi_j^+(x_2)\rangle}{(\langle \psi_i^-(x_1)\psi_i^+(x_1)\rangle \langle \psi_j^-(x_2)\psi_j^+(x_2)\rangle)^{1/2}}.$$

The anticommutation relations will forbid the existence of non-trivial fully coherent states but $g(x_1, x_2)$ will still be a measure of the tendency of the beam to interfere with itself. However, as the wave equation leads to dispersion due to the electron's finite mass, the correspondence of g with the visibility of fringes in an interference experiment will be valid only for stationary and roughly monoenergetic beams. Experiments using such beams have been carried out by researchers such as Mollenstedt (Klemperer 1972) using an electron interferometer. These experiments reveal only the spread of the momenta in the beam and since this is much larger than the minimum spread which arises from the restriction on the number of particles per mode imposed by the anti-commutation relations, the fermion nature of the electron beam remains unimportant. This will be true unless the average number of electrons per coherence volume becomes comparable with one in the cavity where the electrons are produced. The dispersion provided by the electron mass is vital in explaining non-stationary interference phenomena in beams which have an energy substructure such as in the effect of Schwarz and Hora (Schwarz 1971).

Second-order (coincidence) experiments, such as that performed by Hanbury Brown and Twiss (1957a,b) with photons, have not as yet been performed with electrons, largely due to the restriction imposed by the relation (12) and the difficulty of obtaining sufficient temporal resolution. When these problems have been overcome it is still unlikely that the point detector idealization will hold, forcing a derivation of the coincidence rate to use the more general equation (7) for $P(n, t_1, t_2)$. The most obvious feature of the predictions for the results for such an experiment is a dip to zero in the coincidence rate as the delay time goes to zero, which is a direct consequence of the anticommutation relations. As well as revealing such features of the counting statistics which are unique to fermion beams, the results could yield a great deal of information about processes in the electron source and in any medium through which the beam passes.

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